A NOTE ON THE THREE-WAY BALANCED DESIGNS

By

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1. Introduction

These designs were first introduced by Agarwal (1966a) with number of replication $r=v-1$ where v is the number of treatments. He gave the methods of construction of such designs when v is odd. He also furnished solutions for $v=4$ and 16. Raghavarao (1970) showed the existence of such designs for all v. However he gave trial and hit solutions for $v=6$, 8 and 10. In the present paper a systematic method is given which is applicable to odd as well as even number of treatments.

Raghavarao (1971) defined these designs as arrangement of v symbols in a $v \times v$ square such that—

1.1
- (a) diagonal cells are blanks,
- (b) every symbol occurs at most once in any row or column, and
- (c) the i-th symbol does not occur in the i-th row or the i-th column.

If we can construct a latin square in which the i-th symbol occurs in i-th main diagonal position the design defined above can be obtained easily by deleting the main diagonal symbols.

In section 2 below we give a method of construction of such latin squares. It is clear that the incidence matrices of rows, columns and the treatments are same for such designs. In section 3 it is shown that every elementary contrast of the treatments, rows or the columns is estimated with the same variance so that the design is three-way balanced. In the appendix certain designs constructed by the method have been listed.

2. Method of Construction

Without loss of generality we may take the symbols to be 1, 2, ..., v. Let a_{ij} denote the symbol in the i-th row and the j-th column. Then

$$a_{ij}=i$$

2.1 $$a_{ij}=(i+j) \bmod v \text{ for } j>i;\ j=2, 3, \ldots, v-1$$
$$=(i+j-1) \bmod v \text{ for } j<i$$

and

2.2 $$a_{i1}=(2i-1) \bmod v$$
$$a_{iv}=(2i) \bmod v$$

or

$$a_{i1}=(2i-1) \bmod v \text{ for } i=1, 2, \ldots, p-1 \text{ and } 2p$$
$$=(ek) \bmod v \text{ for } i=p, p+1, \ldots, 2p-1$$

2.3 $$a_{iv}=(ei) \bmod v \text{ for } i=1, 2, \ldots, p-1 \text{ and } 2p$$
$$=(2i-1) \bmod v \text{ for } i=p, p+1, \ldots, 2p-1.$$

According as $v=2p+1$ or $2p$ respectively, where p is an integer.

It is clear that the i-th main diagonal position has the i-th symbol. We have now to show that the method above always leads us to a latin square.

From 2.1, we see that j-th column ($j=2, 3, \ldots, v-1$) contains the symbols:

$j, j+1, \ldots, (2j-1) \bmod v, (2j) \bmod v, (2j+1) \bmod v, \ldots, (j+v-1) \bmod v$
Hence all the symbols occur in the j-th column for $j=2, 3, \ldots, v-1$.

Examining 2.2 and 2.3 we find that the first and last columns also contain all the symbols.

To prove that all the symbols occur in each row we restate the equations 2.1 in the following manner:

2.4 $$a_{ii}=i$$
$$a_{ij}=(i+j-1) \bmod v\ i>j=2, 3, \ldots, i-1.$$
$$=(i+j) \bmod v\ i<j=i+1, i+2, \ldots, v-1.$$

and the proof follows in the similar fashion as for columns.

Hence the method described above always provides us a latin square in which i-th symbol occurs in i, i-th position. Deleting the diagonal symbols we get a design as defined in 1.1.

3. Analysis of the Design

For completion of the paper we describe in brief the analysis of the design. We follow the notations of Agarwal (1966*b*). For our design

$$v=u=u'$$

$$r_i=n_{.j}=n_{j.}$$

$$M=L=N$$

$$M+N'=2\,[E_{vv}-I_v]$$

$$MN'=I_v+(v-2)E_{vv}$$

$$C_{11}=C_{22}=C_{33}=\frac{v(v-3)}{(v-2)}\left[\,1_v-\frac{1}{v}E_{vv}\,\right]$$

Therefore, $C_{ii}^*=\dfrac{(v-2)}{v(v-3)}\,I_v$ for $i=1$, 2 and 3.

Where C^*_{ii} denotes the conditional inverse of C_{ii}.

Hence the design is balanced for treatments, rows, and columns.

4. Summary

A generalised systematic method of constructing three-way balanced designs for $r=v-1$ has been furnished here.

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